Announcements

- HW4: 3-41, 44 and 4-13, 15, 20, 31, ex.4.2
- Quiz 2: Next Friday (Ch.3 & 4)

*** Course Web Page **
http://highenergy.phys.ttu.edu/~slee/2402/
Lecture Notes, HW Assignments,
Schedule for thePhysics Colloquium, etc..

$$k \equiv \frac{2\pi}{\lambda} \qquad \omega \equiv \frac{2\pi}{T} = 2\pi f \qquad \hbar \equiv \frac{h}{2\pi}$$

$$p = \frac{h}{\lambda} = \hbar k$$

$$E = hf = \hbar\omega$$

Last Lecture:

The *probability* to find a particle in an interval Δx, Δt

$$P = \iint dxdt \, |\Psi(x,t)|^2$$

Integrate over Δx, Δt

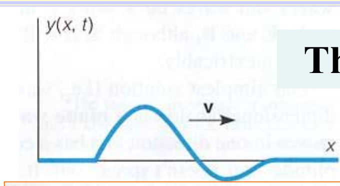
How does the Probability Wave Move?

Equation of Motion for ψ ?

The Free-Particle Schrodinger Wave Equation

[Q] How do we determine the ψ(x,t) of a matter wave?

Waves on a string



The Plane Wave

$$v^2 \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$$

Waves on a string

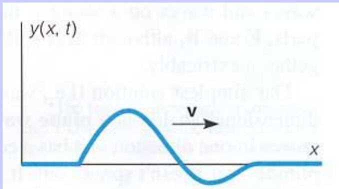


Figure 3.9 A wave disturbance on a string.

$$v^2 \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$$

Solution = function

"Plane Wave"

$$y(x,t) = A\sin(kx - \omega t) \quad \text{where} \quad \frac{\omega}{k} = v$$

The Free-Particle Schrodinger Wave Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

$\psi(x,t)$ **Probability Wave Function**

$\psi^*(x,t)\psi(x,t) = |\psi(x,t)|^2$ **Probability Density**

$\psi(x,t)$ **a complex function**

$$\psi(x,t) \equiv \psi_1(x,t) + i\psi_2(x,t)$$

$$\psi^*(x,t) \equiv \psi_1(x,t) - i\psi_2(x,t)$$

Complex Conjugate

$$i \equiv \sqrt{-1}$$

$$i^2 = -1$$

Probability Density = ?

$$\psi(x,t) \equiv \psi_1(x,t) + i\psi_2(x,t)$$

$$\psi^*(x,t) \equiv \psi_1(x,t) - i\psi_2(x,t)$$

$$i = \sqrt{-1}$$

Probability Density

$$|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t) =$$

$$= \psi_1\psi_1 - ii\psi_2\psi_2 - i\psi_1\psi_2 + i\psi_2\psi_1 =$$

$$= \psi_1\psi_1 + \psi_2\psi_2$$

$$|\psi(x,t)|^2 = \psi_1^{\,2}(x,t) + \psi_2^{\,2}(x,t)$$

4.3 The Free-Particle Schrodinger Eq.

The Plane Wave

[Solution] -- complex exponential

The Plane Wave

$$\Psi(x,\ t) = Ae^{i(kx-\omega t)}$$

$$\Psi(x,t) = \begin{cases} Ae^{i(kx-\omega t)} \\ A\cos(kx-\omega t) + iA\sin(kx-\omega t) \\ \Psi_1(x,t) + i\Psi_2(x,t) \end{cases}$$

Is the Plane Wave a solution of the Schrodinger Equation?

$$\Psi(x,\ t) = Ae^{i(kx-\omega t)}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,\ t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,\ t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 Ae^{i(kx-\omega t)}}{\partial x^2} \stackrel{?}{=} i\hbar\frac{\partial Ae^{i(kx-\omega t)}}{\partial t}$$

$$-\frac{\hbar^2}{2m}(ik)^2 Ae^{i(kx-\omega t)} = i\hbar(-i\omega)Ae^{i(kx-\omega t)}$$

$$\frac{\hbar^2k^2}{2m} = \hbar\omega$$

Is the Plane Wave a solution of the Schrodinger Equation?

$$\Psi(x,\ t) = Ae^{i(kx-\omega t)}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,\ t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,\ t)}{\partial t}$$

$$\frac{\hbar^2k^2}{2m} = \hbar\omega$$

YES

$$\frac{p^2}{2m} = E$$

$$E = \frac{(mv)^2}{2m} = \frac{mv^2}{2}$$

The Magnitude of a Plane Wave

$$\Psi(x,\ t) = Ae^{i(kx-\omega t)}$$

$$|\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t) = [Ae^{-i(kx-\omega t)}]\,[Ae^{+i(kx-\omega t)}] = A^2$$

$$|\Psi(x,t)| = A$$

Constant in space and time!

The Magnitude of a Plane Wave

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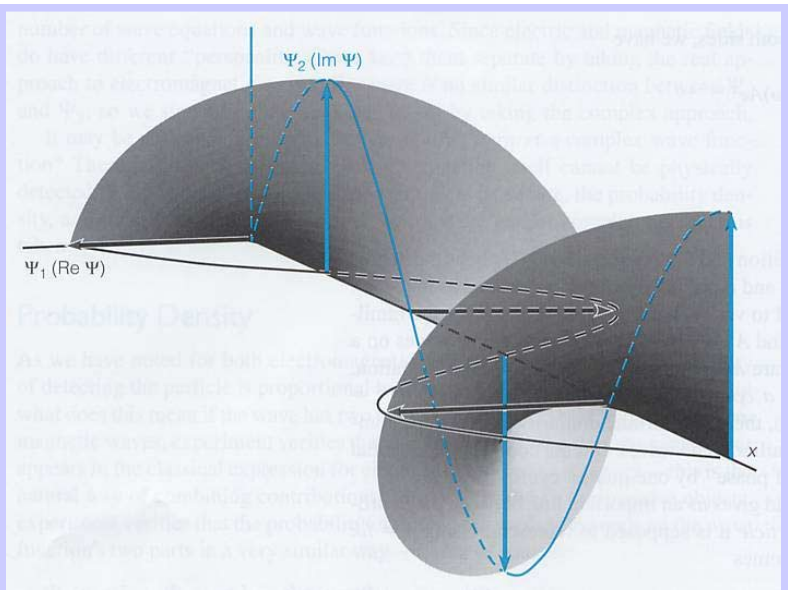
Constant in space and time!

Constant Probability Density



The Schrödinger equation is based upon energy. The simplest solution is a plane wave, a complex exponential with two sinusoidal parts. The wave's magnitude and the probability density vary neither in time nor in position.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,\ t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,\ t)}{\partial t}$$

$$\frac{\hbar^2k^2}{2m} = \hbar\omega \qquad \frac{p^2}{2m} = E$$

Kinetic energy

$$\Psi(x,\ t) = Ae^{i(kx-\omega t)}$$

Plane wave

$$|\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t) = [Ae^{-i(kx-\omega t)}]\,[Ae^{+i(kx-\omega t)}] = A^2$$

$$|\Psi(x,t)| = A$$

= constant (x,t)